EXERCISES Worksheet 3.7B

Finding Limits

In Exercises 1–6, use l'Hôpital's Rule to evaluate the limit. Then evaluate the limit using a method studied in Chapter 2.

1. $\lim_{x\to 2} \frac{x-2}{x^2-4}$
2. $\lim_{x\to 0} \frac{\sin 5x}{x}$
3. $\lim_{x\to \infty} \frac{5x^2-3x}{7x^2+1}$
4. $\lim_{x\to 1} \frac{x^3-1}{4x^3-x-3}$
5. $\lim_{x\to 0} \frac{1-\cos x}{x^2}$
6. $\lim_{x\to \infty} \frac{2x^2+3x}{x^3+x+1}$

Applying l'Hôpital's Rule

Use l'Hôpital's rule to find the limits in Exercises 7–46.

7. $\lim_{x\to 2} \frac{x-2}{x^2-4}$
8. $\lim_{x\to -5} \frac{x^2-25}{x+5}$
9. $\lim_{t\to -3} \frac{t^3-4t+15}{t^2-t-12}$
10. $\lim_{t\to 1} \frac{t^3-1}{4t^3-t-3}$
11. $\lim_{x\to \infty} \frac{5x^3-2x}{7x^3+3}$
12. $\lim_{x\to \infty} \frac{x-8x^2}{12x^2+5x}$
13. $\lim_{t\to 0} \frac{\sin t^2}{t}$
14. $\lim_{t\to 0} \frac{\sin 5t}{2t}$
15. $\lim_{x\to 0} \frac{8x^2}{\cos x - 1}$
16. $\lim_{x\to 0} \frac{\sin x - x}{x^3}$
17. $\lim_{\theta\to \pi/2} \frac{2\theta-\pi}{\cos(2\pi-\theta)}$
18. $\lim_{\theta\to -\pi/3} \frac{3\theta+\pi}{\sin(\theta+(\pi/3))}$
19. $\lim_{\theta\to \pi/2} \frac{1-\sin\theta}{1+\cos 2\theta}$
20. $\lim_{x\to 1} \frac{x-1}{\ln x - \sin \pi x}$
21. $\lim_{x\to 0} \frac{x^2}{\ln(\sec x)}$
22. $\lim_{x\to \pi/2} \frac{\ln(\csc x)}{(x-(\pi/2))^2}$
23. $\lim_{t\to 0} \frac{t(1-\cos t)}{t-\sin t}$
24. $\lim_{t\to 0} \frac{t\sin t}{1-\cos t}$
25. $\lim_{x\to(\pi/2)^-} \left(x-\frac{\pi}{2}\right)\sec x$
26. $\lim_{x\to(\pi/2)^-} \left(\frac{\pi}{2}-x\right)\tan x$
27. $\lim_{\theta\to 0} \frac{3^{\sin\theta}-1}{\theta}$
28. $\lim_{\theta\to 0} \frac{(1/2)^\theta - 1}{\theta}$
29. $\lim_{x\to 0} \frac{x2^x}{2^x-1}$
30. $\lim_{x\to 0} \frac{3^x-1}{2^x-1}$
31. $\lim_{x\to \infty} \frac{\ln(x+1)}{\log_2 x}$
32. $\lim_{x\to \infty} \frac{\log_2 x}{\log_3(x+3)}$
33. $\lim_{x\to 0^+} \frac{\ln(x^2+2x)}{\ln x}$
34. $\lim_{x\to 0^+} \frac{\ln(e^x-1)}{\ln x}$
35. $\lim_{y\to 0} \frac{\sqrt{5y+25}-5}{y}$
36. $\lim_{y\to 0} \frac{\sqrt{ay+a^2}-a}{y}$, $a>0$
37. $\lim_{x\to \infty} (\ln 2x - \ln(x+1))$
38. $\lim_{x\to 0^+} (\ln x - \ln \sin x)$
39. $\lim_{h\to 0} \frac{\sin(a+h)-\sin a}{h}$
40. $\lim_{x\to 0^+} \left(\frac{3x+1}{x}-\frac{1}{\sin x}\right)$
41. $\lim_{x\to 1^+} \left(\frac{1}{x-1}-\frac{1}{\ln x}\right)$
42. $\lim_{x\to 0^+} (\csc x - \cot x + \cos x)$
43. $\lim_{\theta\to 0} \frac{\cos\theta - 1}{e^\theta - \theta - 1}$
44. $\lim_{h\to 0} \frac{e^h-(1+h)}{h^2}$
45. $\lim_{t\to \infty} \frac{e^t+t^2}{e^t-t}$
46. $\lim_{x\to \infty} x^2e^{-x}$

Limits Involving Bases and Exponents

Find the limits in Exercise 47–56.

47. $\lim_{x\to 1^+} x^{1/(1-x)}$
48. $\lim_{x\to 1^+} x^{1/(x-1)}$
49. $\lim_{x\to \infty} (\ln x)^{1/x}$
50. $\lim_{x\to e^+} (\ln x)^{1/(x-e)}$
51. $\lim_{x\to 0^+} x^{-1/\ln x}$
52. $\lim_{x\to \infty} x^{1/\ln x}$
53. $\lim_{x\to \infty} (1+2x)^{1/(2\ln x)}$
54. $\lim_{x\to 0} (e^x+x)^{1/x}$
55. $\lim_{x\to 0^+} x^x$
56. $\lim_{x\to 0^+} \left(1+\frac{1}{x}\right)^x$

Theory and Applications

L'Hôpital's Rule does not help with the limits in Exercises 57–60. Try it—you just keep on cycling. Find the limits some other way.

57. $\lim_{x\to \infty} \frac{\sqrt{9x+1}}{\sqrt{x+1}}$
58. $\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}}$
59. $\lim_{x\to(\pi/2)^-} \frac{\sec x}{\tan x}$
60. $\lim_{x\to 0^+} \frac{\cot x}{\csc x}$

61. Which one is correct, and which one is wrong? Give reasons for your answers.

    a. $\lim_{x\to 3} \frac{x-3}{x^2-3} = \lim_{x\to 3} \frac{1}{2x} = \frac{1}{6}$

    b. $\lim_{x\to 3} \frac{x-3}{x^2-3} = \frac{0}{6} = 0$